The quantized Hall effect in the presence of resistance fluctuations

E. Peled and D. Shahar
Department of Condensed Matter Physics, Weizmann Institute, Rehovot 76100, Israel

Y. Chen
Department of Electrical Engineering, Princeton University, Princeton, New Jersey 08544, USA

D. L. Sivco and A. Y. Cho
Bell Laboratories, Lucent Technologies, 600 Mountain Avenue, Murray Hill, New Jersey 07974, USA

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We present an experimental study of mesoscopic, two-dimensional electronic systems at high magnetic fields. Our samples, prepared from a low-mobility InGaAs/InAlAs wafer, exhibit reproducible, sample specific, resistance fluctuations. Focusing on the lowest Landau level we find that, while the diagonal resistivity displays strong fluctuations, the Hall resistivity is free of fluctuations and remains quantized at its $\nu = 1$ value, h/e^2. This is true also in the insulating phase that terminates the quantum Hall series. These results extend the validity of the semicircle law of conductivity in the quantum Hall effect to the mesoscopic regime.

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For small, mesoscopic samples, the conductivity in the Quantum Hall (QH) regime exhibits reproducible, sample specific, fluctuations [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12]. Although quite interesting by themselves, the greatest benefit in studying these fluctuations is that they can provide clues to the nature of the mechanism responsible for conduction in the QH regime.

There are several known mechanisms that can lead to such fluctuations. Fluctuations as a result of the modification of the interference between many possible electron paths across the sample are at the heart of the theory of Universal Conductance Fluctuations (UCF) [13]. Although this theory is not expected to be valid at high magnetic fields (B), modified UCF theories have been suggested [14, 15] to take into account the influence of B on the electron trajectories, and several experiments were interpreted in terms of UCF at high B [2, 3].

A different point of view has been suggested by two recent experimental studies: Cobden *et al.* [4], and Machida *et al.* [5] have found that in the QH regime fluctuation patterns follow straight lines in the magnetic field–carrier-density plane. These lines appear to be parallel to integer Landau level (LL) filling-factor lines. These results were attributed to charging of electron puddles in the sample [4], or to changes in a compressible-strip network configuration [5].

Another possible source of fluctuations, expected to dominate when B is high enough such that only a small number of LL are occupied, is resonant tunneling [16]. In this process fluctuations arise when an electron scatters from one edge of the sample to the other through a bulk impurity. This model was found to be consistent with observed high-B fluctuations [6], and has been used in measurements of fractional charge [7, 8]. Applying this model to the case when only the lowest LL is occupied, Jain and Kivelson [16] argued that the fluctuations will be limited to the diagonal resistivity, ρ_{xx}, leaving the Hall resistivity, ρ_{xy}, quantized. This conjecture was contended by Bütikker [17]. Experiments have so far shown that fluctuations in ρ_{xx} are accompanied by fluctuations in ρ_{xy} [1, 2, 6], although these experiments were limited to relatively lower B's, where more than one LL participates in the conduction ($\nu \geq 2$).

In this letter we report on an experimental study of the resistivity of mesoscopic samples in the QH regime, focusing mainly on the lowest LL. We find that, while ρ_{xx} may exhibit large fluctuations, ρ_{xy} remains nearly fluctuation-free and quantized at its $\nu = 1$ QH value, h/e^2. This is true even beyond the transition to the insulating phase that terminates the QH series.

Our data were obtained from two samples, T2Ga and T2C, wet-etched from the same InGaAs/InAlAs wafer that contained, after illumination with an LED, a two-dimensional electronic system in a 200 Å quantum well. We defined a Hall-bar geometry with lithographic widths of $W = 10$ μm for sample T2Ga and $W = 2$ μm for sample T2C, and voltage-probe separation of $L = 2 \times W$, maintaining an identical aspect ratio. The samples were cooled in a dilution refrigerator with a base T of 12 mK. The mobility and density of sample T2Ga were $\mu = 22,000$ cm^2/Vsec and $n_s = 1.5 \cdot 10^{11}$ cm^{-2}. Sample T2C was cooled down twice, with $\mu = 23,850$ cm^2/Vsec, $n_s = 1.83 \cdot 10^{11}$ cm^{-2} and $\mu = 13,700$ cm^2/Vsec, $n_s = 1 \cdot 10^{11}$ cm^{-2} at the first (T2Cc) and second (T2Ci) cool-downs, respectively. Four-probe measurements were done using standard AC lock-in techniques with excitation currents of 0.1 − 1 nA and frequencies of 1.7 − 11 Hz. Temperatures below 30 mK are nominal, since the resistivity does not change in the T range 12 − 30 mK.

In Fig. 1 we plot ρ_{xx} and ρ_{xy} for sample T2Cc, taken

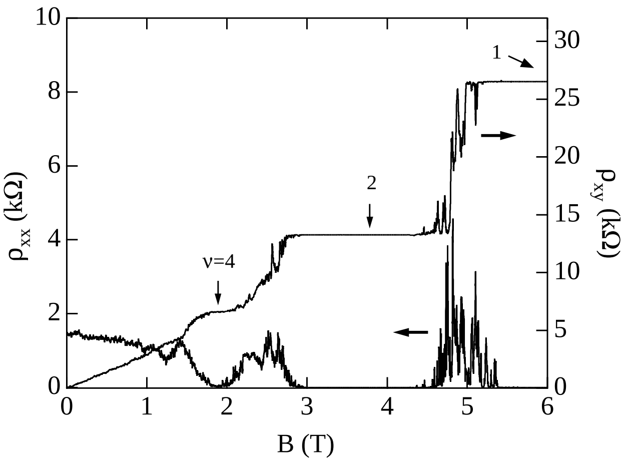

FIG. 1: ρ_{xx} and ρ_{xy} vs. B for sample T2Cc, T = 12 mK. QH plateaus are designated by their filling factor.

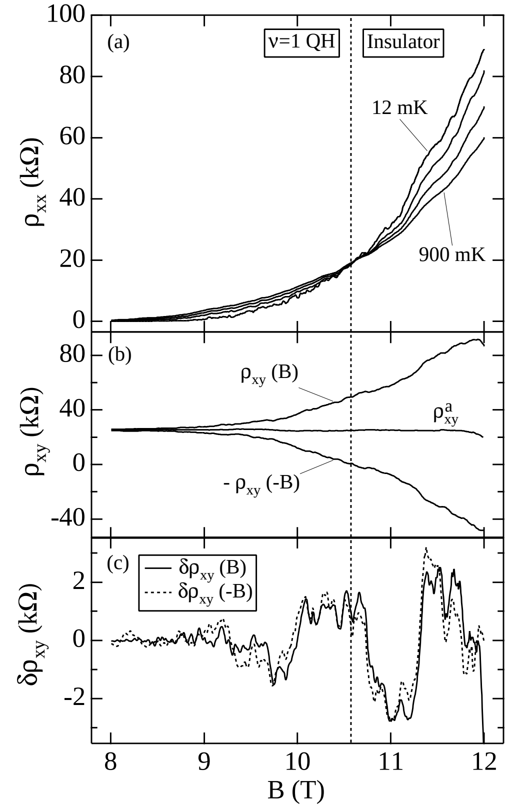

FIG. 2: The resistivity of sample T2Ga at $\nu < 1$. (a) ρ_{xx} traces. Temperatures are 12, 500, 700, and 900 mK. The crossing point of the traces is at $B_c = 10.62$ T ($\nu_c = 0.58$), indicated by the dotted vertical line. (b) $\rho_{xy}(B)$, $-\rho_{xy}(-B)$, and the antisymmetric part, ρ^a_{xy}, at $T = 16$ mK. (c) Comparison of the $\pm B$ fluctuations of ρ_{xy}. The fluctuation patterns are very similar, and overlap to within 92%. This symmetry leads to much smaller fluctuations in ρ^a_{xy}.

over a broad range of B. Although the sample is relatively small ($W = 2$ μm), integer QH plateaus in ρ_{xy} (designated by their filling factor) and the associated minima in ρ_{xx} are clearly observed. Due to the low mobility of our samples, fractional QH features are not seen down to the lowest T. In addition to the QH features, ρ_{xx} and ρ_{xy} exhibit reproducible fluctuations, which maintain their pattern as long as the sample is kept cold, attesting to the mesoscopic nature of our samples. The fluctuations decrease in magnitude when T is increased and have a similar pattern when switching to voltage probes that are on the opposite side of the Hall-bar (for ρ_{xx} measurements) or to a neighbouring pair of voltage probes (for ρ_{xy} measurements). At the low-B range, we associate the fluctuations with the theory of UCF. According to this theory, the coherence length (L_ϕ) can be deduced from the amplitude and correlation-B of the fluctuations. L_ϕ in our samples is found to be between 1.2 to 3 μm at base T, varying from sample to sample and between cooldowns. The larger sample, T2Ga, also exhibits reproducible fluctuations, but of reduced amplitude, due to averaging of subunits of size L_ϕ over the sample area. At $T = 12$ mK and near $B = 0$ the rms of the fluctuations in samples T2Cc and T2Ga is 35 and 9.6 Ω, respectively, corresponding to conductivity fluctuations $\delta\sigma = 0.54$ and 0.067 e^2/h. As expected, the reproducible fluctuations are not limited to the low-B range of Fig. 1, and are present over the entire B range of our measurements. At the high-B range the fluctuations appear on top of the usual QH features, both in ρ_{xx} and in ρ_{xy}, as seen in previous studies [1, 2, 3, 4, 5, 6, 7, 8, 9].

In the remainder of this letter we will focus on data taken at $\nu < 1$, where only the lowest LL contributes to the transport. In Fig. 2 we examine the resistivity obtained from sample T2Ga, in the B range of 8–12 T, corresponding to $\nu = 0.775$–0.517. We begin by focusing on ρ_{xx} traces taken at T =12, 500, 700, and 900 mK in Fig. 2(a). Two different transport regimes can be distinguished according to the T-dependence of ρ_{xx}: the QH liquid regime (low-B side), where ρ_{xx} increases with increasing T, and the insulating regime, (high-B side), where ρ_{xx} decreases with T. The transition B, $B_c = 10.62$ T ($\nu_c = 0.58$), where ρ_{xx} is T-independent, is indicated by the dotted vertical line. The value of ρ_{xx} at the transition, $\rho_{xxc} = 0.77$ h/e^2, is in agreement with previous experimental results where $\rho_{xxc} \sim h/e^2$ [18, 19, 20, 21]. The ρ_{xx} fluctuations here are much larger than at low-B, and have an amplitude as high as 2,840 Ω on the insulating side.

Before we proceed with the main result of our work, which stems from the analysis of the ρ_{xy} data, a word of caution is in order. An experimental determination of ρ_{xy} in the lowest LL is difficult, because an admixture of ρ_{xx} into the data is unavoidable. This is especially important in the insulating regime where ρ_{xx} is large, and even

a small fraction of ρ_{xx} admixture can result in a significant change in the measured ρ_{xy}. The traditional way to overcome this problem is by using the B-symmetry of the resistivity components: ρ_{xx} is expected to be symmetric in B, while ρ_{xy} is antisymmetric. The true ρ_{xy} is obtained, in principle, by an antisymmetrization of the measured $\rho_{xy}(B)$ and $\rho_{xy}(-B)$: $\rho^a_{xy} = \frac{1}{2}[\rho_{xy}(B) - \rho_{xy}(-B)]$. In practice, because the admixture is a result of both contact misalignment and current nonuniformities in the sample, ρ_{xx} itself is not entirely B-symmetric, limiting the accuracy with which ρ_{xy} can be determined.

We now turn to the analysis of the ρ_{xy} data at $\nu < 1$. In Fig. 2(b) we plot $\rho_{xy}(B)$, $-\rho_{xy}(-B)$ and the antisymmetrized ρ^a_{xy} taken at $T = 16$ mK. As in previous works [21, 22, 23, 24], we find that $\rho_{xy}(B)$ and $\rho_{xy}(-B)$ show a strong, symmetric, B-dependence that is canceled out of ρ^a_{xy}, resulting in a Hall coefficient that remains nearly constant and quantized at its $\nu = 1$ value, h/e^2. This holds into the insulating phase, which has been termed the quantized Hall insulator (QHI) [23]. The novelty in our data is that, while $\rho_{xy}(\pm B)$ include fluctuations, they are symmetric in B and therefore do not appear in ρ^a_{xy}. To demonstrate this symmetry more clearly, we compare in Fig. 2(c) the positive and negative B fluctuation patterns, $\delta\rho_{xy}(\pm B)$, after subtracting a smooth background from the original traces. $\delta\rho_{xy}(\pm B)$ are very similar in shape and magnitude, and overlap to within 92% [25]. The maximum amplitude of the $\rho_{xy}(\pm B)$ fluctuations is 2,825 Ω, while that of ρ^a_{xy} is 735 Ω. We attribute these remaining ρ_{xy} fluctuations to an asymmetric component of ρ_{xx} in sample T2Ga. Measurements of $\rho_{xx}(\pm B)$ reveal an asymmetric component whose magnitude is consistent with the fluctuations of ρ^a_{xy}. We stress that even if sample inhomogeneities were not present, obtaining perfectly reproducible resistivity measurements at opposite B polarities is not practical due to the stringent requirements this will place on T stability when the B field is swept over nearly 20 T. This problem becomes even more severe in light of the strong T-dependence of ρ_{xx} in the insulating phase.

In order to enhance the fluctuating part of the data, we repeated our $\nu < 1$ measurements with the smaller ($W = 2$ μm) sample, T2Ci, which was fabricated with utmost care to reduce contact misalignments to the minimum. The results are shown in Fig. 3, where we plot ρ_{xx} and ρ^a_{xy} vs. B taken at $T = 100$, 295, 500, and 698 mK. We start by examining the ρ_{xx} data, which now appear to be dominated by fluctuations that are as large as 188,000 Ω on the insulating side. Nevertheless, by fitting a smooth curve through the data, the average ρ_{xx} can be determined and the QH and insulating regimes properly identified (see inset). The fitted curves cross at $B_c = 7.34$ T ($\nu_c = 0.572$) and $\rho_{xxc} = 0.67\ h/e^2$.

In stark contrast with the ρ_{xx} data, ρ^a_{xy} appears to be free of fluctuations. An upper-bound estimate of the magnitude of the ρ^a_{xy} fluctuations is 160–220 times less

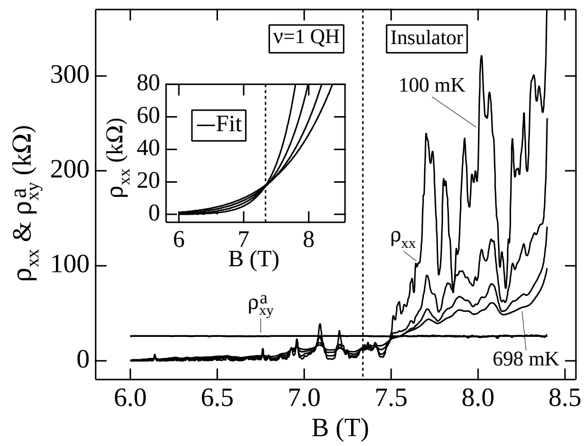

FIG. 3: ρ_{xx} and ρ^a_{xy} of sample T2Ci at high B. $T = 100$, 295, 500, and 698 mK. The large ρ_{xx} fluctuations make it difficult to identify the transition point. Inset: we use the crossing point of the smooth ρ_{xx}-fit curves [33] to identify B_c: $B_c = 7.34$ T ($\nu_c = 0.572$), $\rho_{xxc} = 0.67\ h/e^2$.

than the amplitude of $\delta\rho_{xx}$ at similar background values. Comparing these data to the data obtained from the 10 μm sample, we see that reducing the sample size by a factor of 5 resulted in an increase of $\delta\rho_{xx}$ by nearly 70 while $\delta\rho_{xy}$ increased by only a factor of 2.

We now turn to a discussion of the consequences of our results. So far we have shown that for $\nu < 1$ the two resistivity components of our samples show dissimilar behaviour: ρ_{xx} displays large reproducible fluctuations while ρ_{xy} remains close to its quantized value and shows only insignificant fluctuations.

Of the models mentioned above pertaining to fluctuations in the QH regime only the resonant-tunneling model specifically deals with the behaviour of ρ_{xy}. Our measurements of a fluctuation-free, quantized, ρ_{xy} are consistent with this model.

There are two additional models for conduction in the QH regime that predict a quantized ρ_{xy}, although they do not directly address the properties of the mesoscopic fluctuations of the resistivity. In the first model, Ruzin and his collaborators [26, 27] treat the electron system in the QH transition region as a random mixture of two phases, and find a semicircle relation for the conductivity components, σ_{xx} and σ_{xy}. For the QH-insulator transition, this relation is:

$$\sigma^2_{xx} + \left(\sigma_{xy} - \frac{e^2}{2h}\right)^2 = \left(\frac{e^2}{2h}\right)^2,$$

which is mathematically equivalent to the quantization of ρ_{xy} [23]. Since we are unable to measure the conductivity components directly, we obtain them by inverting the resistivity tensor, ρ_{xx} and ρ^a_{xy}. In Fig. 4 we plot the resulting σ_{xx} and σ_{xy} of sample T2Ci at the QH-insulator transition. In contrast with the resistivity tensor, both

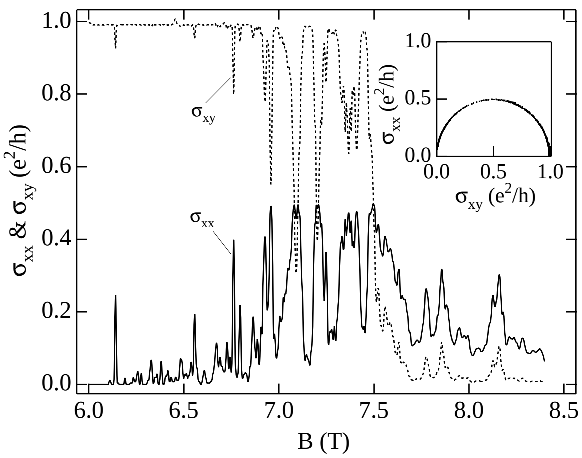

FIG. 4: σ_{xx} and σ_{xy} of sample T2Ci near the QH-insulator transition, calculated by inverting the resistivity tensor. $T = 100$ mK. Inset: The semicircle relation, σ_{xx} vs. σ_{xy}.

σ_{xx} and σ_{xy} display fluctuations, stemming from their mutual dependence on ρ_{xx} as well as on ρ_{xy}^a. Nevertheless, σ_{xx} and σ_{xy} obey the semicircle relation, as shown in the inset of Fig. 4, indicating the special correlation that exists between the fluctuations of the conductivity-tensor components. Our work extends the validity of the semicircle relation to the mesoscopic regime of transport.

Another model pertaining to the quantization of ρ_{xy} near the transition, and in the insulating regime, has been proposed by Shimshoni and Auerbach [28]. They showed that transport in a random Chalker-Coddington-based network of puddles produces a quantized ρ_{xy} when L_ϕ is smaller than the puddle size. However, ρ_{xy} is expected to diverge when L_ϕ is larger than the puddle size [29, 30]. It would be interesting to see how this model can be extended to accommodate samples that exhibit large resistance fluctuations.

Finally, we wish to emphasize that the absence of fluctuations in ρ_{xy} near the QH-insulator transition is not inconsistent with the results of previous experiments (and the present work, see Fig. 1), which show large ρ_{xy} fluctuations that survive the averaging of opposite B directions in the vicinity of the transitions in higher LL's. We recall the mapping [31] that exists between transitions at higher LL's and the 'basic', QH-insulator, transition at the lowest LL: in this mapping one regards the higher transitions as a QH-insulator transition occurring in the presence of a number of full and inert LL's. A simple calculation shows that the ρ_{xy} of higher LL transitions will include components proportional to ρ_{xx} of a QH-insulator transition [32]. We leave for future work a more direct verification of this mapping to the fluctuating part of the resistivity.

To conclude, we have shown that the quantization of the Hall effect in two-dimensional electron systems can be maintained in the mesoscopic regime, even when the diagonal resistivity, ρ_{xx}, is nonzero and exhibits large fluctuations. These results are in agreement with the predictions of Jain and Kivelson [16] and with the semicircle relation for the conductivity components [26, 27].

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